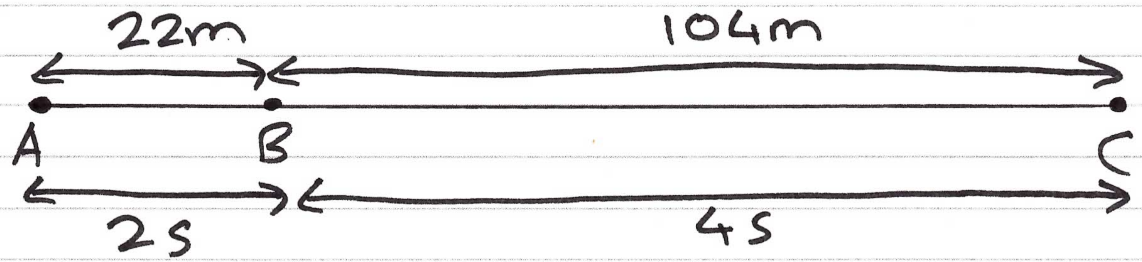
**1.** A car is moving along a straight horizontal road with constant acceleration. There are three points $A$, $B$ and $C$, in that order, on the road, where $AB = 22$ m and $BC = 104$ m. The car takes 2 s to travel from $A$ to $B$ and 4 s to travel from $B$ to $C$.

Find

(i) the acceleration of the car,

(ii) the speed of the car at the instant it passes $A$.

**(7)**




(i) $s = ut + \frac{1}{2}at^2$

$22 = u(2) + \frac{1}{2}a(2)^2$

$22 = 2u + 2a$ ①

Also, $126 = u(6) + \frac{1}{2}a(6)^2$

$126 = 6u + 18a$ ②

To find a, eliminate u by solving simultaneous equations:

$$\begin{array}{lll} 2u + 2a = 22 \quad ① & \times 3 & 6u + 6a = 66 \\ 6u + 18a = 126 \quad ② & \times 1 & 6u + 18a = 126 \end{array} \Bigg\} -$$

$-12a = -60$

$\therefore a = 5$

$\boxed{\text{acceleration of the car} = 5\text{ms}^{-2}}$

(ii) Substitute a into ① for u:

$$2u + 2a = 22$$

$$2u + 2(5) = 22$$

$$2u + 10 = 22$$

$$2u = 12$$

$$\therefore u = 6$$

$\boxed{\text{Speed of car as it passes } A = 6\text{ms}^{-1}}$

## SECTION B: MECHANICS

Unless otherwise indicated, wherever a numerical value of $g$ is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

**Answer ALL questions. Write your answers in the spaces provided.**

**2.** A man throws a tennis ball into the air so that, at the instant when the ball leaves his hand, the ball is 2 m above the ground and is moving vertically upwards with speed $9\,\mathrm{m\,s^{-1}}$

The motion of the ball is modelled as that of a particle moving freely under gravity and the acceleration due to gravity is modelled as being of constant magnitude $10\,\mathrm{m\,s^{-2}}$

The ball hits the ground $T$ seconds after leaving the man's hand.

Using the model, find the value of $T$.

**(4)**

$u = 9 \quad a = -10 \quad s = -2$

$s = ut + \frac{1}{2}at^2$

$-2 = 9t + \frac{1}{2}(-10)t^2$

$5t^2 - 9t - 2 = 0$

$(5t+1)(t-2) = 0$

$t = -\frac{1}{5}$ n/a $\quad t = 2$

$T = 2$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Answer ALL questions. Write your answers in the spaces provided.**

**3.** At time $t = 0$, a parachutist falls vertically from rest from a helicopter which is hovering at a height of 550 m above horizontal ground.

The parachutist, who is modelled as a particle, falls for 3 seconds before her parachute opens.

While she is falling, and before her parachute opens, she is modelled as falling freely under gravity.

The acceleration due to gravity is modelled as being $10\,\text{m}\,\text{s}^{-2}$.

(a) Using this model, find the speed of the parachutist at the instant her parachute opens.
**(1)**

When her parachute is open, the parachutist continues to fall vertically.

Immediately after her parachute opens, she decelerates at $12\,\text{m}\,\text{s}^{-2}$ for 2 seconds before reaching a constant speed and she reaches the ground with this speed.

The total time taken by the parachutist to fall the 550 m from the helicopter to the ground is $T$ seconds.

(b) Sketch a speed-time graph for the motion of the parachutist for $0 \leqslant t \leqslant T$.
**(2)**

(c) Find, to the nearest whole number, the value of $T$.
**(5)**

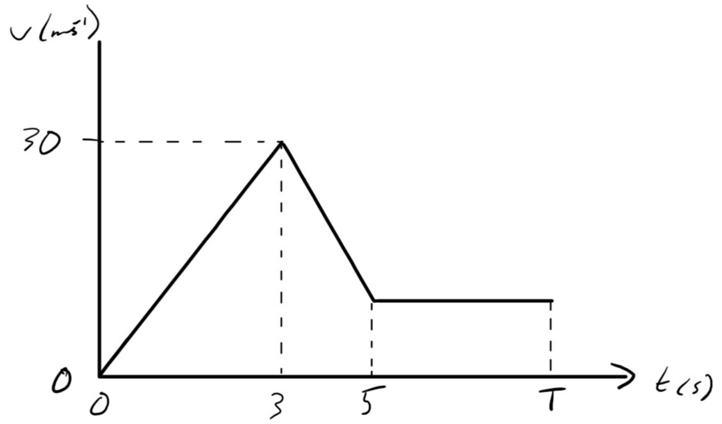
In a refinement of the model of the motion of the parachutist, the effect of air resistance is included before her parachute opens and this refined model is now used to find a new value of $T$.

(d) How would this new value of $T$ compare with the value found, using the initial model, in part (c)?
**(1)**

(e) Suggest one further refinement to the model, apart from air resistance, to make the model more realistic.
**(1)**

a. $v = u + at = 0 + 10 \times 3 = 30\,\text{ms}^{-1}$

b.



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c. $s = \frac{1}{2}(u+v)t$

acceleration section : $s = \frac{1}{2}(0+30) \times 3 = 45$ m

deceleration section : $s = \frac{1}{2}(30+6) \times 2 = 36$ m

constant section : $s = vt = 6(T-5)$

$\Sigma s = 550$ : $550 = 45 + 36 + 6T - 30$

$6T = 499$

$T = 83$

d. New T would be larger

e. Include effect of wind

or

Include dimensions + spin of parachutist

or

Use a more accurate value for g

or

Allow for parachutist not falling vertically

Only one of these needed for the 1 mark

**4.** At time $t = 0$, a small ball is projected vertically upwards with speed $U\,\text{m s}^{-1}$ from a point $A$ that is 16.8 m above horizontal ground.

The speed of the ball at the instant immediately before it hits the ground for the first time is $19\,\text{m s}^{-1}$

The ball hits the ground for the first time at time $t = T$ seconds.

The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

Using the model,

(a) show that $U = 5$ **(2)**

(b) find the value of $T$, **(2)**

(c) find the time from the instant the ball is projected until the instant when the ball is 1.2 m below $A$. **(4)**

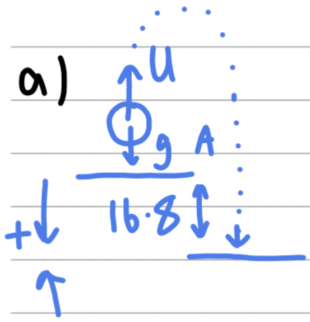
(d) Sketch a velocity-time graph for the motion of the ball for $0 \leqslant t \leqslant T$, stating the coordinates of the start point and the end point of your graph. **(2)**

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of $U$.

(e) State, with a reason, how this new value of $U$ would compare with the value found in part (a), using the initial unrefined model. **(1)**

(f) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic. **(1)**

a)




define axis

no air resistance

opposite direction to U @ start

S 16.8   U −u   V 19   A g   T

when ball returns to start point, its velocity is U downwards

$\hookrightarrow v^2 = u^2 + 2as$

$19^2 = (-U)^2 + 2 \times 10 \times 16.8$

$361 = U^2 + 336$

$U^2 = 25 \therefore U = 5$

b) $V = u + at \Rightarrow 19 = -5 + 10T$

remember the direction of positive velocity

↳ down is positive, so when ball moves upwards, its velocity is negative

$24 = 10T$

$T = 2.4$

c) S 1.2 U -5 V A g T t

$S = ut + \frac{1}{2}at^2$

$1.2 = -5t + \frac{1}{2} \times 10 \times t^2$

$1.2 = -5t + 5t^2$

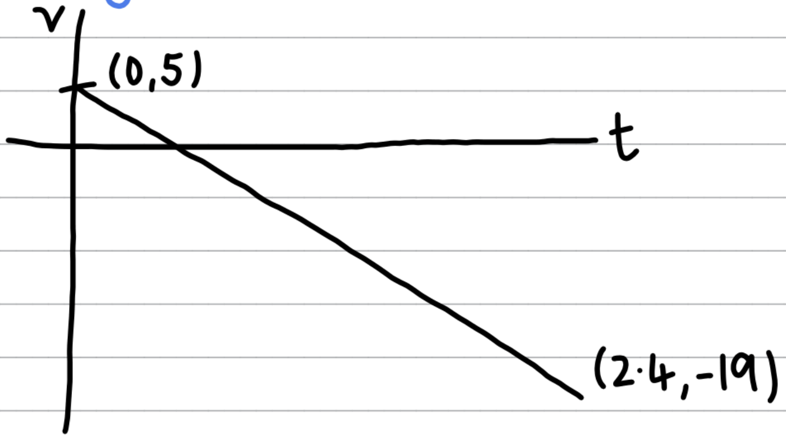
$5t^2 - 5t - 1.2 = 0$

$t = \dfrac{5 \pm \sqrt{5^2 - 4 \times 5 \times -1.2}}{10}$

$= 1.2$ or $-0.2$

t must be $>0$ so $t = 1.2s$

d) v-t graph ⇒ constant acceleration ⇒ constant gradient




(0,5) & (2·4, -19) (can also give (0,-5) & (2·4, +19))

e) air resistance will slow the ball down, so to have a speed of $19\,ms^{-1}$ after a time T, the ball must start with a faster speed ⇒ larger U calculated.

f) e.g. spin, wind effects, use more accurate value of g.

**5.** At time $t = 0$, a small stone is thrown vertically upwards with speed $14.7\,\mathrm{m\,s^{-1}}$ from a point $A$.

At time $t = T$ seconds, the stone passes through $A$, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(a) find the value of $T$,

**(2)**

(b) find the total distance travelled by the stone in the first 4 seconds of its motion.

**(4)**

(c) State one refinement that could be made to the model, apart from air resistance, that would make the model more realistic.

**(1)**

(a) $s = ut + \frac{1}{2}at^2$

displacement from A to A

downward motion has –ve sign

$0 = 14.7T + \frac{1}{2} \times (-9.8) \times T^2$

$0 = 14.7T - \frac{1}{2} \times 9.8 \times T^2$ 1

$0 = 14.7T - 4.9T^2$

$0 = T(14.7 - 4.9T)$

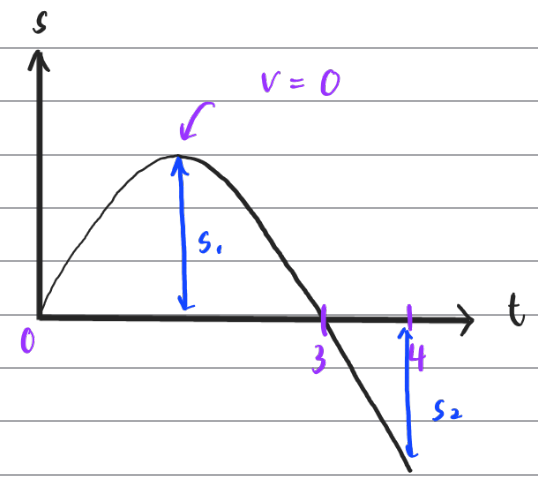
$T = 0$ or $14.7 - 4.9T = 0$

$4.9T = 14.7$

$T = 3$

$\therefore T = 3$ (only solution) 1

(b)



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$v^2 = u^2 + 2as$

$0^2 = (14.7)^2 + 2(-9.8)s_1$

$0 = 216.09 - 19.6s_1$

$19.6s_1 = 216.09$

$s_1 = 11.025$ m (1)

$s = ut + \frac{1}{2}at^2$

$s_2 = 14.7 \times 4 + \frac{1}{2} \times (-9.8)(4)^2$ (1)

$s_2 = 58.8 - 78.4$

upwards + downwards

$= -19.6$

$\therefore s_2 = 19.6$ m

$s_1 \times 2 + s_2$

total distance travelled $= 11.025 \times 2 + 19.6$ (1)

$= 41.65$ m

$\approx 41.7$ m (3sf) (1)

(c) Take into account the dimentions of the stone (e.g. allow for spin) (1)

*[In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively]*

**6.** A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point $O$ and is moving due north with speed $0.6 \text{ m s}^{-1}$.

Relative to $O$, the position vector of the boat at time $t$ seconds is $\mathbf{r}$ metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j}) \text{ m s}^{-1}$.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$.
**(2)**

(b) Find $\mathbf{r}$ in terms of $t$.
**(2)**

(c) Find the value of $t$ when the boat is north-east of $O$.
**(3)**

(d) Find the value of $t$ when the boat is moving in a north-east direction.
**(3)**

a) We want to find the acceleration from $t = 0s$ and $t = 15s$.

$\underline{r} = X$

$\underline{u} = 0.6\underline{j} \text{ ms}^{-1}$

$\underline{v} = (10.5\underline{i} - 0.9\underline{j}) \text{ ms}^{-1}$

$\underline{a} = ?$

$t = 15s$ (Scaler)

$\underline{v} = \underline{u} + \underline{a}t$

$10.5\underline{i} - 0.9\underline{j} = 0.6\underline{j} + 15\underline{a}$ 1

$10.5\underline{i} - 0.9\underline{j} - 0.6\underline{j} = 15\underline{a}$

$10.5\underline{i} - 1.5\underline{j} = 15\underline{a}$

$\underline{a} = (0.7\underline{i} - 0.1\underline{j}) \text{ ms}^{-2}$ 1

b) We want to find $\underline{r}$ in terms of $t$.

$\underline{r} = ?$

$\underline{u} = 0.6\underline{j}$

$\underline{V} = 10.5\underline{i} - 0.9\underline{j}$

$\underline{a} = 0.7\underline{i} - 0.1\underline{j}$

$t = ?$

$\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2$ 1

$\underline{r} = 0.6\underline{j}\,t + \frac{1}{2}(0.7\underline{i} - 0.1\underline{j})t^2$ 1

c) North East => the vectors $\underline{i}$ and $\underline{j}$ will be equal in the expression for displacement. 1

$$\Rightarrow \quad \underline{r} = 0.6\underline{j}t + \tfrac{1}{2}(0.7\underline{i} - 0.1\underline{j})t^2$$

$$= 0.6\underline{j}t + (0.35\underline{i} - 0.05\underline{j})t^2$$

$$= 0.6\underline{j}t + 0.35\underline{i}t^2 - 0.05\underline{j}t^2$$

$$\underline{r} = t(0.6\underline{j} + 0.35\underline{i}t - 0.05\underline{j}t)$$

$\underline{j}$ component $\Rightarrow$ $0.6 - 0.05t = 0.35t$ ← $\underline{i}$ component 1

$\Rightarrow 0.6 = 0.4t$

$\Rightarrow \underline{\underline{t = 1.5s}}$ 1

d) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = 0.6\underline{j} + (0.7\underline{i} - 0.1\underline{j})t$ 1

Now, we want to set the $\underline{i}$ and $\underline{j}$ components of $\underline{v}$ equal to each other.

$$\Rightarrow \quad \underline{v} = 0.6\underline{j} + 0.7\underline{i}t - 0.1\underline{j}t$$

$\Rightarrow \quad 0.6 - 0.1t = 0.7t$ 1 $\Rightarrow \quad 0.6 = 0.8t$

$\underline{\underline{t = 0.75s}}$ 1

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.*]

**7.** A particle $P$ moves with constant acceleration.
At time $t = 0$, the particle is at $O$ and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$
At time $t = 2$ seconds, $P$ is at the point $A$ with position vector $(7\mathbf{i} - 10\mathbf{j})\,\text{m}$.

(a) Show that the magnitude of the acceleration of $P$ is $2.5\,\text{m s}^{-2}$

**(4)**

At the instant when $P$ leaves the point $A$, the acceleration of $P$ changes so that $P$ now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\,\text{m s}^{-2}$

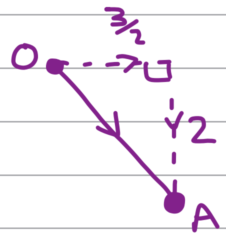
At the instant when $P$ reaches the point $B$, the direction of motion of $P$ is north east.

(b) Find the time it takes for $P$ to travel from $A$ to $B$.

**(4)**

a) $\underline{s} = 7\underline{i} - 10\underline{j}$
$\underline{u} = 2\underline{i} - 3\underline{j}$
$\underline{v} = /$
$\underline{a} = ?$
$t = 2$

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$ – (1)
$7\underline{i} - 10\underline{j} = 2(2\underline{i} - 3\underline{j}) + \frac{1}{2}\underline{a}(2)^2$
$7\underline{i} - 10\underline{j} = 4\underline{i} - 6\underline{j} + 2\underline{a}$
$2\underline{a} = 3\underline{i} - 4\underline{j}$

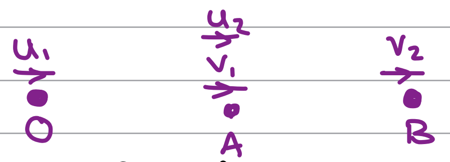



$\underline{a} = \left(\frac{3}{2}\underline{i} - 2\underline{j}\right)\text{ms}^{-2}$ – (1)

$|\underline{a}| = \sqrt{\left(\frac{3}{2}\right) + (2)^2}$ – (1)

$|\underline{a}| = 2.5\,\text{ms}^{-2}$ – (1)

b)

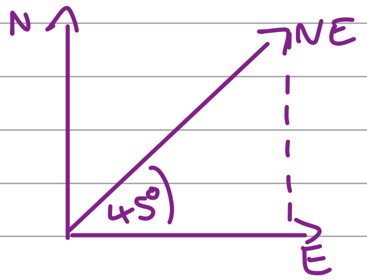



From O to A:
$\underline{s} = 7\underline{i} - 10\underline{j}$
$\underline{u} = 2\underline{i} - 3\underline{j}$
$\underline{v} = ?$
$\underline{a} = \frac{3}{2}\underline{i} - 2\underline{j}$
$t = 2$

$\underline{v} = \underline{u} + \underline{a}t$ – (1)
$\underline{v} = 2\underline{i} - 3\underline{j} + 2\left(\frac{3}{2}\underline{i} - 2\underline{j}\right)$
$\underline{v} = 2\underline{i} - 3\underline{j} + 3\underline{i} - 4\underline{j}$

$\underline{v} = 5\underline{i} - 7\underline{j}$ – (1)

b) From A to B:




$\tan 45 =$

for velocity:

$\underset{\sim}{i}$ component = $\underset{\sim}{j}$ component

$\underset{\sim}{s} = /$
$\underset{\sim}{u} = 5\underset{\sim}{i} - 7\underset{\sim}{j}$
$\underset{\sim}{v} = ?$
$\underset{\sim}{a} = 4\underset{\sim}{i} + 8.8\underset{\sim}{j}$
$t = ?$

$\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t$ - 1
$\underset{\sim}{v} = 5\underset{\sim}{i} - 7\underset{\sim}{j} + (4\underset{\sim}{i} + 8.8\underset{\sim}{j})t$
$\underset{\sim}{v} = (5+4t)\underset{\sim}{i} + (8.8t - 7)\underset{\sim}{j}$

$5 + 4t = 8.8t - 7$
$5 = 4.8t - 7$

$t = 2.5$ seconds - 1

**8.** A particle $P$ moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, $P$ is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of $P$ at time $t = 2$ seconds.

**(2)**

At time $t = 0$, the position vector of $P$ relative to a fixed origin $O$ is $(\mathbf{i} + \mathbf{j})\,\text{m}$.

(b) Find the position vector of $P$ relative to $O$ at time $t = 3$ seconds.

**(2)**

(a) $u = 4i$

$a = 2i - 3j$

$t = 2$

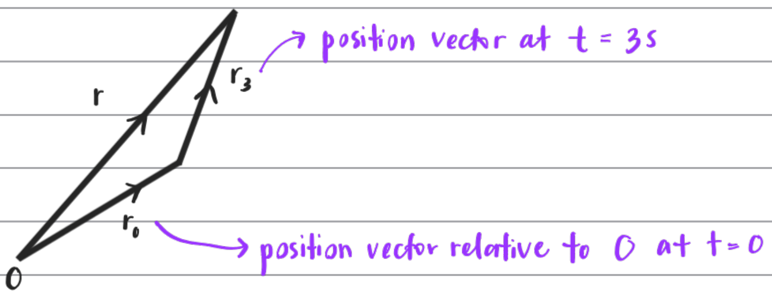
$v = u + at$

$v = 4i + (2i - 3j)2$ (1)

$= 4i + 4i - 6j$

$\therefore \; v = 8i - 6j$ (1)

(b)




$r_0 = i + j$

$r = ut + \frac{1}{2}at^2$

$r_3 = 4i \times 3 + \frac{1}{2} \times (2i - 3j) \times (3)^2$ (1)

$= 12i + 9i - 13.5j$

$= 21i - 13.5j$

$r = r_0 + r_3$

$= i + j + 21i - 13.5j$

$r = 22i - 12.5j$ (1)